Reply to `Comment on ``Garden of Eden states in a traffic model revisited'' '

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COMMENT

Reply to 'Comment on "Garden of Eden states in a traffic model revisited"'

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Abstract

A consistent theory is presented. The Kolmogorov conditions are satisfied with the state-dependent normalizations $\mathcal{N}_{\tau,\tau'}$. However, such a consistent theory does not provide a better description to the flow–density relation. Eliminating the Garden of Eden states as done in the paradisical mean-field theory will not succeed when one goes beyond the simple mean-field theory.

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In our previously published letter [1] we studied the flow–density relation in the Nagel–Schreckenberg traffic model [2], which has attracted much attention recently. We investigated the combination of two successful approximations: the 2-cluster approximation and the paradisical mean-field theory. We found that each single approximation gives a better result than their combination. We also discussed the underlying physics leading to such an unexpected result. We concluded with that the probability interpretation has not been treated rigorously in the paradisical mean-field theory. This conclusion was questioned in a Comment by Schadschneider and Schreckenberg [3]. The authors argued that the surprising result is due to the inconsistency of the combined theory, which violates the elementary Kolmogorov conditions. They also suggested that a consistent theory satisfying the Kolmogorov conditions would certainly lead to an improvement in describing the flow–density relation.

In the paradisical mean-field theory, the dynamically forbidden states (Garden of Eden states) are eliminated deliberately by deleting the corresponding terms appeared in the mean-field expressions. A new normalization is then introduced to compensate for the reduction of phase space. In [1], only a single normalization $\mathcal{N}$ is introduced to renormalize the probability. Thus the Kolmogorov conditions cannot be satisfied, as pointed out correctly in the Comment. To satisfy the Kolmogorov conditions, one has to relax the constraint that the 2-site probabilities $P_{\tau,\tau'}$ were normalized by the same factor. In [3], the state-dependent normalizations $\mathcal{N}_{\tau,\tau'}$ were suggested to provide a remedy. We find that the Kolmogorov conditions can be fully restored by allowing the seven normalizations $\mathcal{N}_{\tau,\tau'}$ to have four different values. For example, we allow P_{x2}, P_{1x}, and P_{11} to be renormalized differently, i.e., $\mathcal{N}_{x2}$, $\mathcal{N}_{1x}$, and $\mathcal{N}_{11}$ are taken as independent normalizations. And the fourth normalization is applied to P_{xx}, P_{x1}, P_{2x}, and P_{21}, which all involve the paradisical state $\langle\langle 1xx2\rangle\rangle$. Thus a consistent mean-field theory is obtained by imposing the constraints $\mathcal{N}_{xx} = \mathcal{N}_{x1} = \mathcal{N}_{2x} = \mathcal{N}_{21}$. The results are listed in table 1.

Table 1. Results of a consistent mean-field theory. The Kolmogorov conditions are satisfied. The normalizations $\mathcal{N}_{\tau,\tau'}$ are constrained to $\mathcal{N}_{xx} = \mathcal{N}_{x1} = \mathcal{N}_{2x} = \mathcal{N}_{21}$.

ρ	P_{xx} ($\mathcal{N}_{xx}$)	P_{x1} ($\mathcal{N}_{x1}$)	P_{x2} ($\mathcal{N}_{x2}$)	P_{1x} ($\mathcal{N}_{1x}$)	P_{2x} ($\mathcal{N}_{2x}$)	P_{11} ($\mathcal{N}_{11}$)	P_{21} ($\mathcal{N}_{21}$)
0.1	0.817 88 (1.006 67)	0.009 77 (1.006 67)	0.072 35 (1.004 41)	0.010 59 (1.013 16)	0.071 53 (1.006 67)	0.017 06 (1.180 24)	0.000 82 (1.006 67)
0.2	0.652 93 (1.020 86)	0.030 85 (1.020 86)	0.116 22 (1.014 37)	0.035 89 (1.020 45)	0.111 18 (1.020 86)	0.047 89 (1.151 38)	0.005 04 (1.020 86)
0.3	0.499 48 (1.037 00)	0.057 54 (1.037 00)	0.142 98 (1.026 46)	0.071 88 (1.022 86)	0.128 65 (1.037 00)	0.085 14 (1.109 92)	0.014 34 (1.037 00)
0.4	0.359 59 (1.048 23)	0.084 89 (1.048 23)	0.155 52 (1.035 87)	0.114 07 (1.021 67)	0.126 34 (1.048 23)	0.130 41 (1.065 79)	0.029 18 (1.048 23)
0.5	0.237 77 (1.047 99)	0.106 95 (1.047 99)	0.155 28 (1.037 26)	0.154 76 (1.018 39)	0.107 46 (1.047 99)	0.189 96 (1.030 16)	0.047 81 (1.047 99)
0.6	0.139 83 (1.035 99)	0.116 91 (1.035 99)	0.143 26 (1.029 30)	0.182 01 (1.013 61)	0.078 16 (1.035 99)	0.274 74 (1.009 60)	0.065 10 (1.035 99)
0.7	0.069 95 (1.019 99)	0.109 66 (1.019 99)	0.120 39 (1.017 12)	0.183 14 (1.008 08)	0.046 91 (1.019 99)	0.396 46 (1.001 93)	0.073 48 (1.019 99)
0.8	0.027 04 (1.007 79)	0.085 06 (1.007 79)	0.087 90 (1.007 03)	0.151 75 (1.003 42)	0.021 21 (1.007 79)	0.560 34 (1.000 21)	0.066 70 (1.007 79)
0.9	0.005 82 (1.001 59)	0.046 94 (1.001 59)	0.047 24 (1.001 51)	0.088 97 (1.000 75)	0.005 21 (1.001 59)	0.763 79 (1.000 01)	0.042 03 (1.001 59)

The Kolmogorov conditions are now satisfied. However, the resultant flow is still lower than that of the 2-cluster approximation and much lower than that of the paradisical mean-field theory. Basically the flow–density relation reported in [1] has been reproduced. We have tried other options of constraining the seven normalizations to four different values, e.g., $\mathcal{N}_{x1} = \mathcal{N}_{1x}$, $\mathcal{N}_{x2} = \mathcal{N}_{2x}$, and $\mathcal{N}_{11} = \mathcal{N}_{21}$. Similar results are observed. Imposing the Kolmogorov conditions to make the theory consistent does not lead to an improvement in describing the flow–density relation. In fact, such a result is not surprising. As we pointed out in [1], the success of the paradisical mean-field theory lies in the elimination of the states $\langle\langle 12\rangle\rangle$ and $\langle\langle 22\rangle\rangle$, which have more weighting on the lower speed. Thus a renormalization will shift weighting toward a higher speed and the flow will increase considerably, with respect to the results of simple mean-field theory, i.e., 1-cluster approximation. When combined with the 2-cluster approximation, the paradisical states to be eliminated becomes $\langle\langle 1x2\rangle\rangle$, $\langle\langle 2x2\rangle\rangle$, and $\langle\langle 1xx2\rangle\rangle$, which have more weighting on the higher speed. On the contrary, the procedure of elimination and renormalization will shift weighting back to a lower speed and the flow will decrease as observed. Thus stands our conclusion in [1]. Eliminating the paradisical states as done in the paradisical mean-field theory will not give a better description to the flow–density relation when one goes beyond the 1-cluster approximation.

References

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